Digression: "Sparrow" (Prolog) Syntax for DL

1. Person
2. Female
3. Woman $\equiv$ Person $\sqcap$ Female
4. Man $\equiv$ Person $\sqcap$ $\neg$Woman
5. Mother $\equiv$ Woman $\sqcap$ $\exists hasChild$.Person
6. Father $\equiv$ Man $\sqcap$ $\exists hasChild$.Person
7. Parent $\equiv$ (Father $\sqcup$ Mother)
8. Grandmother $\equiv$ Mother $\sqcap$ $\exists hasChild$.Parent
9. Wife $\equiv$ Woman $\sqcap$ $\exists hasHusband$.Man
10. MotherWithoutDaughter $\equiv$ Mother $\sqcap$ $\forall hasChild$.$\neg$Woman

```
:- op(600, xfx, sub).
:- op(600, xfx, eqv).

:- op(550, xfy, or).

:- op(500, xfy, and).

:- op(400, xfy, some).
:- op(400, xfy, only).

:- op(350, fx, neg).
```

Sparrow "Grammar" and "Parser"

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'Person'.
'Female'.
'Woman' eqv 'Person' and 'Female'.
'Man' eqv 'Person' and neg 'Woman'.
'Mother' eqv 'Woman' and 'hasChild' some 'Person'.
'Father' eqv 'Man' and 'hasChild' some 'Person'.
'Parent' eqv 'Father' and 'Mother'.
'Grandmother' eqv 'Mother' and 'hasChild' some 'Parent'.
'Wife' eqv 'Woman' and 'hasHusband' some 'Man'.
'MotherWithoutDaughter' eqv 'Mother' and 'hasChild' only neg 'Woman'.
```

Example in Sparrow Syntax

Introduction to DL: Syntax and Semantics of $\mathcal{ALC}$

Semantics given by means of an **interpretation** $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$:

Constructor	Syntax	Example	Semantics
atomic concept	A	Human	$A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
atomic role	R	likes	$R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
For C, D concepts and R a role name			
conjunction	$C \sqcap D$	Human $\sqcap$ Male	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$
disjunction	$C \sqcup D$	Nice $\sqcup$ Rich	$C^{\mathcal{I}} \cup D^{\mathcal{I}}$
negation	$\neg C$	$\neg$ Meat	$\Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
exists restrict.	$\exists R.C$	$\exists$has-child.Human	$\{x \mid \exists y.\langle x, y\rangle \in R^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}\}$
value restrict.	$\forall R.C$	$\forall$has-child.Blond	$\{x \mid \forall y.\langle x, y\rangle \in R^{\mathcal{I}} \Rightarrow y \in C^{\mathcal{I}}\}$

Source: Description Logics Tutorial, Ian Horrocks and Ulrike Sattler, ECAI-2002, Lyon, France, July 23rd, 2002

Introduction to DL: Other DL Constructors

Constructor	Syntax	Example	Semantics
number restriction	$(\geq n\ R)$	$(\geq 7$ has-child$)$	$\{x \mid \lvert\{y.\langle x, y\rangle \in R^{\mathcal{I}}\}\rvert \geq n\}$
	$(\leq n\ R)$	$(\leq 1$ has-mother$)$	$\{x \mid \lvert\{y.\langle x, y\rangle \in R^{\mathcal{I}}\}\rvert \leq n\}$
inverse role	R^{-}	has-child^{-}	$\{\langle x, y\rangle \mid \langle y, x\rangle \in R^{\mathcal{I}}\}$
trans. role	R^{*}	has-child*	$(R^{\mathcal{I}})^{*}$
concrete domain	$u_1, \ldots, u_n.P$	h-father-age, age. $>$	$\{x \mid \langle u_1^{\mathcal{I}}, \ldots, u_n^{\mathcal{I}}\rangle \in P\}$
etc.			

Many different DLs/DL constructors have been investigated

Source: Description Logics Tutorial, Ian Horrocks and Ulrike Sattler, ECAI-2002, Lyon, France, July 23rd, 2002

Introduction to DL: Knowledge Bases: TBoxes

For terminological knowledge: **TBox** contains

Concept definitions $A \doteq C$ (A a concept name, C a complex concept)

Father $\doteq$ Man $\sqcap$ $\exists$has-child.Human

Human $\doteq$ Mammal $\sqcap$ $\forall$has-child$^-$.Human

$\leadsto$ introduce macros/names for concepts, can be (a)cyclic

Axioms $C_1 \sqsubseteq C_2$ (C_i complex concepts)

$\exists$favourite.Brewery $\sqsubseteq$ $\exists$drinks.Beer

$\leadsto$ restrict your models

An interpretation $\mathcal{I}$ **satisfies**

a **concept definition** $A \doteq C$ iff $A^\mathcal{I} = C^\mathcal{I}$

an **axiom** $C_1 \sqsubseteq C_2$ iff $C_1^\mathcal{I} \subseteq C_2^\mathcal{I}$

a **TBox** $\mathcal{T}$ iff $\mathcal{I}$ satisfies all definitions and axioms in $\mathcal{T}$

$\leadsto$ $\mathcal{I}$ is a **model** of $\mathcal{T}$

Source: Description Logics Tutorial, Ian Horrocks and Ulrike Sattler, ECAI-2002, Lyon, France, July 23rd, 2002

Introduction to DL: Knowledge Bases: ABoxes

For assertional knowledge: **ABox** contains

Concept assertions $a : C$ (a an individual name, C a complex concept)

John : Man $\sqcap$ $\forall$has-child.(Male $\sqcap$ Happy)

Role assertions $\langle a_1, a_2\rangle : R$ (a_i individual names, R a role)

$\langle$John, Bill$\rangle$: has-child

An interpretation $\mathcal{I}$ **satisfies**

a **concept assertion** $a : C$ iff $a^\mathcal{I} \in C^\mathcal{I}$

a **role assertion** $\langle a_1, a_2\rangle : R$ iff $\langle a_1^\mathcal{I}, a_2^\mathcal{I}\rangle \in R^\mathcal{I}$

an **ABox** $\mathcal{A}$ iff $\mathcal{I}$ satisfies all assertions in $\mathcal{A}$

$\leadsto$ $\mathcal{I}$ is a **model** of $\mathcal{A}$

Source: Description Logics Tutorial, Ian Horrocks and Ulrike Sattler, ECAI-2002, Lyon, France, July 23rd, 2002

Introduction to DL: Basic Inference Problems

Subsumption: $C \sqsubseteq D$ — Is $C^\mathcal{I} \subseteq D^\mathcal{I}$ in all interpretations $\mathcal{I}$?

w.r.t. TBox $\mathcal{T}$: $C \sqsubseteq_\mathcal{T} D$ — Is $C^\mathcal{I} \subseteq D^\mathcal{I}$ in all models $\mathcal{I}$ of $\mathcal{T}$?

$\leadsto$ structure your knowledge, compute taxonomy

Consistency: Is C consistent w.r.t. $\mathcal{T}$? — Is there a model $\mathcal{I}$ of $\mathcal{T}$ with $C^\mathcal{I} \neq \emptyset$?

of ABox $\mathcal{A}$: Is $\mathcal{A}$ consistent? — Is there a model of $\mathcal{A}$?

of KB $(\mathcal{T},\mathcal{A})$: Is $(\mathcal{T},\mathcal{A})$ consistent? — Is there a model of both $\mathcal{T}$ and $\mathcal{A}$?

Inference Problems are **closely related**:

$C \sqsubseteq_\mathcal{T} D$ iff $C \sqcap \neg D$ is inconsistent w.r.t. $\mathcal{T}$,

(no model of $\mathcal{I}$ has an instance of $C \sqcap \neg D$)

C is consistent w.r.t. $\mathcal{T}$ iff not $C \sqsubseteq_\mathcal{T} A \sqcap \neg A$

$\leadsto$ **Decision Procdures for consistency (w.r.t. TBoxes) suffice**

Source: Description Logics Tutorial, Ian Horrocks and Ulrike Sattler, ECAI-2002, Lyon, France, July 23rd, 2002

Relationship with Other Logical Formalisms: First Order Predicate Logic

Most DLs are decidable fragments of FOL: Introduce

a unary predicate A for a concept name A
a binary relation R for a role name R

Translate complex concepts C, D as follows:

$$\begin{aligned}
t_x(A) &= \mathrm{A}(x), & t_y(A) &= \mathrm{A}(y),\\
t_x(C \sqcap D) &= t_x(C) \wedge t_x(D), & t_y(C \sqcap D) &= t_y(C) \wedge t_y(D),\\
t_x(C \sqcup D) &= t_x(C) \vee t_x(D), & t_y(C \sqcup D) &= t_y(C) \vee t_y(D),\\
t_x(\exists R.C) &= \exists y.\mathrm{R}(x,y) \wedge t_y(C), & t_y(\exists R.C) &= \exists x.\mathrm{R}(y,x) \wedge t_x(C),\\
t_x(\forall R.C) &= \forall y.\mathrm{R}(x,y) \Rightarrow t_y(C), & t_y(\forall R.C) &= \forall x.\mathrm{R}(y,x) \Rightarrow t_x(C).
\end{aligned}$$

A TBox $\mathcal{T} = \{C_i \doteq D_i\}$ is translated as

$$\Phi_{\mathcal{T}} = \forall x. \bigwedge_{1 \le i \le n} t_x(C_i) \Leftrightarrow t_x(D_i)$$

Source: Description Logics Tutorial, Ian Horrocks and Ulrike Sattler, ECAI-2002, Lyon, France, July 23rd, 2002

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Relationship with Other Logical Formalisms: First Order Predicate Logic II

C is consistent iff its translation $t_x(C)$ is satisfiable,

C is consistent w.r.t. $\mathcal{T}$ iff its translation $t_x(C) \wedge \Phi_{\mathcal{T}}$ is satisfiable,

$C \sqsubseteq D$ iff $t_x(C) \Rightarrow t_x(D)$ is valid

$C \sqsubseteq_{\mathcal{T}} D$ iff $\Phi_t \Rightarrow \forall x.(t_x(C) \Rightarrow t_x(D))$ is valid.

- ⇝ $\mathcal{ALC}$ is a fragment of FOL with 2 variables (L2), known to be decidable
- ⇝ $\mathcal{ALC}$ with inverse roles and Boolean operators on roles is a fragment of L2
- ⇝ further adding number restrictions yields a fragment of C2 (L2 with "counting quantifiers"), known to be decidable
- ✦ in contrast to most DLs, adding transitive roles (binary relations/ transitive closure operator) to L2 leads to **undecidability**
- ✦ many DLs (like many modal logics) are fragments of the **Guarded Fragment**
- ✦ most DLs are less complex than L2: L2 is NExpTime-complete, most DLs are in ExpTime

Source: Description Logics Tutorial, Ian Horrocks and Ulrike Sattler, ECAI-2002, Lyon, France, July 23rd, 2002

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