Digression:
“Sparrow” (Prolog) Syntax for DL

1. Person
2. Female
3. Woman ⊑ Person ⊑ Female
4. Man ⊑ Person ⊑ Woman
5. Mother ⊑ Woman ⊑ hasChild ⊑ Person
6. Father ⊑ Man ⊑ hasChild ⊑ Person
7. Parent ⊒ (Father ⊑ Mother)
8. Grandmother ⊑ Mother ⊑ hasChild ⊑ Grandchild
9. Wife ⊑ Woman ⊑ hasChild ⊑ Man
10. Mother/Granddaughter ⊑ Mother ⊑ hasChild ⊑ Woman

Sparrow “Grammar” and “Parser”

Example in Sparrow Syntax

Introduction to DL: Syntax and Semantics of ALC

Semantics given by means of an interpretation \( \mathcal{I} = (\Delta^I, \cdot^I) \):

<table>
<thead>
<tr>
<th>Constructor</th>
<th>Syntax</th>
<th>Example</th>
<th>Semantics</th>
</tr>
</thead>
<tbody>
<tr>
<td>atomic concept</td>
<td>( A )</td>
<td>Human</td>
<td>( A^I \subseteq \Delta^I )</td>
</tr>
<tr>
<td>atomic role</td>
<td>( R )</td>
<td>likes</td>
<td>( R^I \subseteq \Delta^I \times \Delta^I )</td>
</tr>
<tr>
<td>conjunction</td>
<td>( C \sqcap D )</td>
<td>Human ( \sqcap ) Male</td>
<td>( C^I \cap D^I )</td>
</tr>
<tr>
<td>disjunction</td>
<td>( C \sqcup D )</td>
<td>Nice ( \sqcup ) Rich</td>
<td>( C^I \cup D^I )</td>
</tr>
<tr>
<td>negation</td>
<td>( \neg C )</td>
<td>~ Meat</td>
<td>( \Delta^I \setminus C^I )</td>
</tr>
<tr>
<td>existential restriction</td>
<td>( \exists R.C )</td>
<td>has-child ( \cdot ) Human</td>
<td>( { x \mid \exists y \cdot (x, y) \in R^I \land y \in C^I } )</td>
</tr>
<tr>
<td>value restriction</td>
<td>( \forall R.C )</td>
<td>has-child ( \cdot ) Blend</td>
<td>( { x \mid \forall y \cdot (x, y) \in R^I \Rightarrow y \in C^I } )</td>
</tr>
</tbody>
</table>

Source: Description Logics Tutorial, Ian Horrocks and Ulrike Sattler, ECAL 2002, Lyon, France, July 23rd, 2002

Introduction to DL: Other DL Constructors

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<tr>
<th>Constructor</th>
<th>Syntax</th>
<th>Example</th>
<th>Semantics</th>
</tr>
</thead>
<tbody>
<tr>
<td>number restriction</td>
<td>( (\geq n \cdot R) )</td>
<td>( \geq n \cdot \text{has-child} )</td>
<td>( { x \mid \left( { y \mid (x, y) \in R^I } \right) \geq n } )</td>
</tr>
<tr>
<td>inverse role</td>
<td>( R^\text{-} )</td>
<td>( \text{has-mother} )</td>
<td>( { x \mid \left( { y \mid (x, y) \in R^I } \right) \leq n } )</td>
</tr>
<tr>
<td>trans. role</td>
<td>( R^* )</td>
<td>( \text{has-child} )</td>
<td>( { &lt;x, y&gt; \mid (x, y) \in R^I } )</td>
</tr>
<tr>
<td>concrete domain etc.</td>
<td>( w_1, \ldots, w_n \cdot P )</td>
<td>n-children, age, etc.</td>
<td>( { \langle x, { w_1, \ldots, w_n } \rangle \in P } )</td>
</tr>
</tbody>
</table>

Many different DLs/DL constructors have been investigated

Source: Description Logics Tutorial, Ian Horrocks and Ulrike Sattler, ECAL 2002, Lyon, France, July 23rd, 2002
For terminological knowledge: TBox contains

Concept definitions
- \( A \equiv C \) (A a concept name, C a complex concept)
- Father \( \equiv \) Man \( \land \) hasChild Human

Human \( \equiv \) Mammal \( \land \) hasChild \_Human

\( \sim \) introduce names/labels for concepts, can be (acyclic):

Axioms
- \( C_1 \subseteq C_2 \) (C complex concepts)
- \( \text{instance} \) \subseteq \( \text{instances} \)

\( \sim \) restrict your models

An interpretation \( \mathcal{I} \) satisfies:

- a concept definition \( A \equiv C \) \( \iff \) \( A^\mathcal{I} = C^\mathcal{I} \)
- an axiom \( C_1 \subseteq C_2 \) \( \iff \) \( C_1^\mathcal{I} \subseteq C_2^\mathcal{I} \)

A TBox \( \mathcal{I} \) satisfies all definitions and axioms in \( \mathcal{I} \)

\( \mathcal{T} \) is a model of \( \mathcal{I} \)

For assertional knowledge: ABox contains

Concept assertions
- \( a \equiv C \) (a a individual name, C a complex concept)
- John \( \equiv \) Man \( \land \) hasChild (Mother \( \land \) Happy)

Role assertions
- \( (a_1, a_2) \equiv R \) (a_1 a_2 individual names, R a role)

\( (\text{John}, \text{Mary}) \) has child

An interpretation \( \mathcal{I} \) satisfies:

- a concept assertion \( a \equiv C \) \( \iff \) \( a^\mathcal{I} \in C^\mathcal{I} \)
- a role assertion \( (a_1, a_2) \equiv R \) \( \iff \) \( \{a_1, a_2\} \subseteq R^\mathcal{I} \)

A TBox \( \mathcal{I} \) satisfies all assertions in \( \mathcal{A} \)

\( \sim \) \( \mathcal{I} \) is a model of \( \mathcal{A} \)

For subsumption:

- \( C \subseteq D \) \( \iff \) \( C^\mathcal{I} \subseteq D^\mathcal{I} \) in all interpretations \( \mathcal{I} \)

w.r.t. TBox \( \mathcal{T} \):

- \( C \subseteq D \) \( \iff \) \( C^\mathcal{I} \subseteq D^\mathcal{I} \) in all models \( \mathcal{I} \) of \( \mathcal{T} \)

\( \sim \) structure your knowledge, compute taxonomy

Consistency:

- Is there a model \( \mathcal{I} \) of \( \mathcal{T} \) with \( C^\mathcal{I} \neq 0 \) of ABox \( \mathcal{A} \)?

Is there a model of \( \mathcal{A} \)?

- Is there a model of \( \mathcal{T} \) and \( \mathcal{A} \)?

Is there a model of both \( \mathcal{T} \) and \( \mathcal{A} \)?

Inference Problems are closely related:

- \( C \subseteq D \) \( \iff \) \( C \models \neg D \) is inconsistent w.r.t. \( \mathcal{T} \)

- \( C \models \neg A \) \( \iff \) \( A \nleftarrow \mathcal{T} \) in an instance of \( C \nleftarrow \mathcal{T} \)

\( \sim \) Decision Procedures for consistency (w.r.t. TBox) suffice
Most DLs are decidable fragments of FOL: Introduce

- a unary predicate $A$ for a concept name $A$
- a binary relation $R$ for a role name $R$

Translating complex concepts $C, D$ as follows:

- $t_c(A) = A(x)$
- $t_c(C \cap D) = t_c(C) \land t_c(D)$
- $t_c(C \cup D) = t_c(C) \lor t_c(D)$
- $t_c(\exists R.C) = \exists x.R(x, y) \land t_c(C)$
- $t_c(\forall R.C) = \forall x.R(x, y) \land t_c(C)$
- $t_c(\forall R.C) = \forall y.R(x, y) \Rightarrow t_c(C)$

A TBox $T = \{ C \equiv D \}$ is translated as

$$\forall \Phi. \bigwedge_{c} t_{c}(C) \Rightarrow t_{c}(D)$$

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$C$ is consistent if its translation $t_c(C)$ is satisfiable,

$C$ is consistent w.r.t. $T$ if its translation $t_c(C) \land \Phi_T$ is satisfiable,

$C \subseteq D$ if $t_c(C) \Rightarrow t_c(D)$ is valid

$C \models D$ if $\Phi_{c} \Rightarrow \forall x.(t_{x}(C) \Rightarrow t_{x}(D))$ is valid.

$\neg ALC$ is a fragment of FOL with 2 variables (L2), known to be decidable

$\neg ALC$ with inverse roles and Binary operators on roles is a fragment of L2

$\neg$ further adding number restrictions yields a fragment of L2

L2 is NEpT-complete, most DLs are in EpT

L2 is in NEpT-complete, most DLs are in EpT